RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 2616

Statistics 4

Tuesday **18 JANUARY 2005** Afternooon 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF12)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer any **three** questions.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The allocation of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is 60.

This question paper consists of 4 printed pages.

1 Technologists are studying equipment for spray-painting large metal surfaces. Minor blemishes called "ripples" occur. The number of ripples per square metre is given by the random variable X having the Poisson distribution with parameter λ.

(i) The equipment is used to spray-paint, independently, three experimental metal sheets, of areas $1\ \mathrm{m}^2$, $4\ \mathrm{m}^2$, $10\ \mathrm{m}^2$. The random variables representing the numbers of ripples are X_1, X_2, X_3 respectively.

Show that

$$\hat{\lambda} = \tfrac{1}{15}(X_1 + X_2 + X_3)$$

is an unbiased estimator of λ. Show further that $\mathrm{Var}(\hat{\lambda}) = \tfrac{1}{15}\lambda$. [8]

(ii) The equipment is then used to spray-paint, independently, n metal sheets each of area $10\ \mathrm{m}^2$. The random variable representing the sample mean number of ripples on such metal sheets is $\overline{Y}$.

Show that $\tfrac{1}{10}\overline{Y}$ is an unbiased estimator of λ and find its variance. [7]

(iii) Deduce that $\tfrac{1}{10}\overline{Y}$ is a better estimator of λ than $\hat{\lambda}$ provided $n \geqslant 2$. If sheets of area $1\ \mathrm{m}^2$ were used instead of sheets of area $10\ \mathrm{m}^2$, how large a sample would be needed so that the sample mean number of ripples, $\overline{Z}$, would be a better estimator of λ than $\hat{\lambda}$? [5]

2 A company operates two call centres for telephone calls from the public. Managers are investigating the service standards at the two centres.

(a) The times spent waiting to be answered are measured automatically by the telephone equipment. For a random sample of 80 calls at the first centre, the mean waiting time is 12.6 seconds and the standard deviation is 2.4 seconds. For a random sample of 90 calls at the second centre, the mean waiting time is 13.9 seconds and the standard deviation is 3.5 seconds. (Standard deviations are as defined using divisor $n - 1$.)

Test at the 1% level of significance whether there is evidence that the true mean waiting times at the two centres differ, stating carefully the null and alternative hypotheses you are testing. Provide a two-sided 95% confidence interval for the true mean difference. [12]

(b) An experienced manager contacts 6 callers to the first centre and 7 callers to the second. Each caller is interviewed in depth over the telephone and asked to arrive at an overall "satisfaction score" combining several aspects of the service. These satisfaction scores are regarded as random samples from corresponding underlying populations for the two centres, and are as follows.

First centre	39.6	36.2	42.4	35.4	42.8	30.6	
Second centre	36.0	34.6	29.4	26.7	42.2	32.3	28.8

(Note that high values indicate good service.)

Use an appropriate non-parametric test, at the 10% level of significance, to examine whether there are, on the whole, any differences between satisfaction scores for the two centres. [8]

3 A security firm believes that good short-term visual memory is an important attribute for its officers, and a psychologist is investigating whether this can be improved by a special coaching programme as part of the initial training of new recruits. Each of a random sample of 12 new recruits takes a standard short-term visual memory test before and after the coaching programme. The results are as follows.

Recruit	A	B	C	D	E	F	G	H	I	J	K	L
Before	52	43	34	46	39	58	51	57	65	63	41	41
After	58	54	56	41	38	62	79	55	72	66	50	49

(a) Use an appropriate Wilcoxon procedure to test whether short-term visual memory has been improved, at the 5% level of significance. [7]

(b) Making an appropriate assumption about underlying Normality, which should be carefully stated, provide an alternative analysis using an appropriate t test, again at the 5% level of significance. [9]

By considering the data, comment briefly and informally on whether the assumption about underlying Normality appears to hold. (You may wish to use a simple diagram.) [4]

4 An office experiences a lack of reliability of its email system when transmitting messages. Many emails are successfully transmitted at the first attempt; others are eventually successfully transmitted, but only after more than one attempt; and others are not successfully transmitted at all. The computer manager thinks there may be an association between the success of transmission and the type of user at the intended destination. Results for a random sample of 300 emails are as follows.

		Type of destination		
		Commercial user	Government department	University
Transmission	Successful at first attempt	100	57	23
	Successful after more than one attempt	21	14	13
	Not successful at all	31	21	20

(i) State the null and alternative hypotheses under examination in the usual χ^2 test applied to this contingency table. [2]

(ii) Carry out the test, at the 10% significance level. [12]

(iii) Discuss your conclusions. [6]

Mark Scheme

Question 1

(i)	We have : $X_1 \sim \text{Poisson}(\lambda)$ $X_2 \sim \text{Poisson}(4\lambda)$ $X_3 \sim \text{Poisson}(10\lambda)$		**M1** might be implicit in sequel
	$\hat{\lambda} = \frac{1}{15}(X_1 + X_2 + X_3)$		**M1** for any attempt to find $E(\hat{\lambda})$
	$\mathrm{E}(\hat{\lambda}) = \frac{1}{15}(\lambda + 4\lambda + 10\lambda)$		**M1** for use of Poisson means
	$= \lambda$		**A1**
	$\therefore \hat{\lambda}$ *is unbiased*		**1**
	$\mathrm{Var}(\hat{\lambda}) = \frac{1}{15^2}\mathrm{Var}(X_1 + X_2 + X_3)$		**M1** for any (reasonable) attempt to find Var
	$= \frac{1}{15^2}(\lambda + 4\lambda + 10\lambda)$		**M1** for use of Poisson variances
	$= \frac{\lambda}{15}$		**A1 -** beware printed answer
			8
(ii)	Now $Y \sim \text{Poisson}(10\lambda)$		
	$\mathrm{E}(\frac{1}{10}\overline{Y}) = \frac{1}{10}\mathrm{E}(\overline{Y}) = \frac{1}{10}\mathrm{E}(Y) = \frac{1}{10}.10\lambda$		**M1**
	$= \lambda$		**A1**
	i.e. unbiased		**1**
	$\mathrm{Var}(\frac{1}{10}\overline{Y}) = \frac{1}{100}\mathrm{Var}(\overline{Y})$		**M1**
	$= \frac{1}{100}\cdot\frac{\mathrm{Var}(Y)}{n}$		**M1**
	$= \frac{1}{100}\cdot\frac{10\lambda}{n} = \frac{\lambda}{10n}$		**M1**, **A1**
			7

(iii)	$\dfrac{\lambda}{10n} < \dfrac{\lambda}{15}$ for $n \geq 2$		**M1**
	ie $\dfrac{1}{10}\overline{Y}$ is better		**E1**
	For $Z \sim$ Poisson (λ), we have $\mathrm{Var}(\overline{Z}) = \dfrac{\lambda}{n}$		**1**
	So would need $n \geq 16$ to be better than $\hat{\lambda}$		**E2** Allow 1 for $n \geq 15$
			5

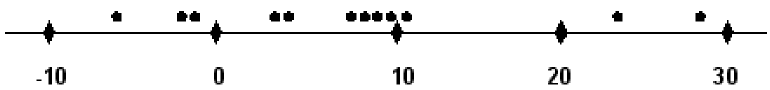
Question 2

(a)	<u>MUST</u> be N (0,1) test and CI for comparing means $H_0 : \mu_1 = \mu_2$ $H_1 : \mu_1 \neq \mu_2$		**1** if <u>both</u> correct. DO <u>NOT</u> allow $\overline{X}_1 = \overline{X}_2$ or similar. Allow verbal statement **1** if μ_1, μ_2 are adequately defined in words (<u>population</u> mean times ……)
	Test statistic is $\dfrac{12\cdot 6 - 13\cdot 9}{\sqrt{\dfrac{(2\cdot 4)^2}{80} + \dfrac{(3\cdot 5)^2}{90}}}$		**M1**
	$\dfrac{-1.3}{\sqrt{0.208\dot{1}}} = \dfrac{-1.3}{0.4562} = -2.84(97)$		**A1**
	Refer to N (0,1) 1% critical point (two-sided) is 2.576 Significant Seems mean waiting times differ		**1** No FT if wrong **1** No FT if wrong **1** **1**
	CI is given by -1.3 ± 1.96 × 0.4562 = -1.3 ± 0.894 = (-2.194, -0.406) **A1** **M1** **B1** **M1**		accept (-2.2, -0.4)
			12
(b)	MUST be Wilcoxon rank-sum test (or Mann-Whitney form thereof). It is convenient, and natural, to rank "top down" Use of Ranks **M1**		[For "bottom-up" rankings W = 55, MW = 34 <u>Upper</u> 5% tail W=55, MW = 34]
	Ranks are: I 4 5 2 7 1 10 II 6 8 11 13 3 9 12		**A1**
	Rank sum (for I) is 29 (Mann-Whitney is 8)		**1**
	Refer to tables of Wilcoxon (or M-W) statistic		**1**
	Lower 5% tail is needed		**1**

	Value for (6,7) is 29 (or 8 if M-W used) Result is significant Seems on the whole there are differences in "satisfaction scores"		**1** **1** **1**
			8

Question 3

(a)	Differences (after – before): $6 \quad 11 \quad 22 \quad -5 \quad -1 \quad 4 \quad 28 \quad -2 \quad 7 \quad 3 \quad 9 \quad 8$ MUST be PAIRED WILCOXON test. Ranks of $\lvert d \rvert$ are $6 \quad 10 \quad 11 \quad \underset{\uparrow}{5} \quad \underset{\uparrow}{1} \quad 4 \quad 12 \quad \underset{\uparrow}{2} \quad 7 \quad 3 \quad 9 \quad 8$ Test statistic is 5 + 1 + 2 = 8 [or 70] Refer to paired Wilcoxon table with n=12 Lower 5% point is 17 [upper is 61] $\therefore$ the observed 8 [or 70] is significant Seems coaching programme has improved short-term visual memory		**M1** **A1** FT if wrong **1** **1** **1** **1** **1**
			7
(b)	MUST be PAIRED COMPARISON t test Normality of <u>differences</u> $\bar{d} = 7.5 \qquad S_{n-1} = 9.5299 \ (S_{n-1}{}^2 = 90.8182)$ Test statistic (for test of $\mu_D = 0$ against $\mu_D > 0$) is $\dfrac{7.5 - 0}{\dfrac{9.5299}{\sqrt{12}}} = 2.72\ (62)$ Refer to t_{11} Upper 5% pt is 1.796 Significant Seems coaching programme has improved short-term visual memory		**1** M1 for use of differences **B1** Accept S_n = 9.1248 ($S_n{}^2$ = 83.85) ONLY if correctly used in sequel **M1 A1** **1** No FT if wrong **1** No FT if wrong **1** **1**
			9

	Look at <u>differences</u> Consider e.g. dotplot  Bulk of data appear OK [assuming no concern about being integers], but the two large upper outliers cast doubt		**M1** M1, or for any other relevant display/discussion of the data **E2** (E0, E1, E2)
			4

Question 4

<table>
<tr><td>(i)</td><td>H_0: no association (between success of transmission and type of destination)
H_1: association</td><td></td><td>1

1</td></tr>
<tr><td colspan="4" align="right">2</td></tr>
<tr><td>(ii)</td><td>O_i

100 57 23 | 180
21 14 13 | 48
31 21 20 | 72
152 92 56 | 300

E_i

91.2(0) 55.2(0) 33.6(0)
24.32 14.72 8.96
36.48 22.08 13.46

Contributions to X^2

0.8491 0.0587 3.3440
0.4532 0.0352 1.8216
0.8232 0.0528 3.2019

$X^2 = 10.63$ (985)
awrt 10.64

Refer to χ_4^2

Upper 10% point is 7.779
Significant
Seems there is association</td><td></td><td>A4 - deduct 1 per error
Must be to this level of accuracy

M1
A2
[give A1 if $\in (10.5, 10.8)$]

2[or zero; FT if wrong, unless ≈ 300]
1
1
1 ← ZERO if $H_0 \leftrightarrow H_1$</td></tr>
<tr><td colspan="4" align="right">12</td></tr>
<tr><td>(iii)</td><td>The key feature is the behaviour of transmission when intended destinations are universities. There are many more “more than one attempt”, and many more “not successful at all”, transmissions than would be expected if there were no association, and many fewer “successful at first attempt” transmissions. There is little or no suggestion of any other associations.</td><td></td><td>E6 (divisible)</td></tr>
<tr><td colspan="4" align="right">6</td></tr>
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Examiner’s Report